Detailed information can be obtained from H. Alfares and A. Al-Amer, "An optimization model for guiding the petrochemical industry development in Saudi Arabia," Engineering Optimization, vol. 34, pp. 671-687, 2002/01/01 2002.

NOMENCLATURE

Following indices, sets, parameters and variables are required in the proposed formulation.

DECISION VARIABLES	X^j	amount of production carried out in process *j*.
	Y^j	binary variable assumes value of one if production level X^j is between l^j and m^j and zero otherwise.
	Z^j	binary variable, assumes a value of one if production level X^j is greater than zero, and zero otherwise.
	L^j, M^j, H^j	represents the proportion of production level *l*, level *m* and level *h* respectively in production of a product using process *j*.
PARAMETERS	i	Product number, i = 1….,*I* (total number of products)
	j	Process number, i = 1….,*J* (total number of processes)
	k	Capacity level indicator
	t	Raw material number, t = 1…,*T* (total number of raw materials)
	q	By-product number, q = 1…,*Q* (total number of by-products)
	b_t^j	Total amount of raw material *t* consumed in process *j*.
	B	Total budget available.
	c_k^j	unit production cost of process *j* for capacity level *k*.
	E^j	per unit export selling price for process j.
	l^j, m^j, h^j	Low, medium and high level production capacity of process *j* respectively.
	R_t	total available amount of raw material *t*.
	v_k^j	investment cost for process *j* at capacity *k*.
	S_q^j	selling price of by-product *q* from process *j*.
	S_i	set of all processes with the main product *i*.
	N_i	number of process in product *i*.
	W_q^j	production quantity of by-product k in process *j*.
	Q^j	number of by-products in process *j*.
	α_q^j	product dependent production capacity of by-product k in process *j*.
	β_q^j	fixed production capacity of by-product *k* in process *j*.

OBJECTIVE

The objective is to maximize the profit (*P*) given in Equation (1) which is the difference between the total revenue obtained by selling the products (and by-products) and the total production cost incurred.

$$Max \quad P = \sum_{j=1}^{J} E^j X^j + \sum_{j=1}^{J} \sum_{q=1}^{Q} \left(S_q^j W_q^j - C^j \right) \qquad (1)$$

CONSTRANTS

$$X^j = l^j L^j + m^j M^j + h^j H^j \qquad \forall j = 1, 2.., J \qquad (2)$$

In Equation (2), X^j is expressed as a linear combination of three variables L^j, M^j and H^j which denotes the amount of product that is produced from process *j*. These three variables depict the proportions of the production levels l^j, m^j and h^j being used.

$$L^j \le Y^j \qquad \forall j = 1,2..,J \qquad (3)$$

$$H^j \le 1 - Y^j \qquad \forall j = 1,2..,J \qquad (4)$$

Equation (3) and (4) ensure that the binary variable Y^j would assume a value of 1 if the amount of production is between l^j and m^j, and a value of 0 if production level is between m^j and h^j.

$$L^j + M^j + H^j = Z^j \qquad \forall j = 1,2..,J \qquad (5)$$

$$X^j \le \lambda Z^j \qquad \forall j = 1,2..,J \qquad (6)$$

In Equation (5) and (6), the sum of L^j, M^j and H^j is restricted to 1 if $X^j > 0$, and equal to 0 if $X^j = 0$.

$$C^j = c_l^j L^j + c_m^j M^j + c_h^j H^j \qquad (7)$$

Equation (7) determines the cost of producing X^j from the process *j* and would be equal to zero if a process is not employed.

$$W_q^j = \alpha_q^j X^j + \beta_q^j Z^j \quad \forall q = 1,2,..Q, j = 1,2,..J \qquad (8)$$

In Equation (8), W_q^j is the total amount of by-product of type *q* that is produced from process *j* and is a linear function of X^j as α_q^j and β_q^j are parameters which are to be known a priori.

$$\sum_{j=1}^{J} b_t^j X^j \le R_t \qquad \forall t = 1,.....,T \qquad (9)$$

Equation (9) restricts that the total amount of raw material of type *t* that is required does not exceed the total available quantity (R_t).

$$\sum_{j=1}^{J} v_l^j L^j + v_m^j M^j + v_h^j H^j \le B \qquad (10)$$

Equation (10) restricts the total investment cost of all the selected processes from exceeding the total budget available (B).

$$\sum_{j \in S_i} Z_j \le N_i \qquad \forall i = 1,.....,I \qquad (11)$$

Equation (11) is the *custom process constraint* (CPC) and limits the number of processes used for the production of each product to a pre-specified value (N_i). If all the values of N_i are restricted to one, it would correspond to the *unique process constraint* (UPC) used in the literature. Equation (1) to (11) comprise the Mixed Integer Linear Programming model for this combinatorial optimization problem.

STRUCTURE

$$\Big[\overbrace{X^1, X^2, X^3,....., X^J}^{\text{Continuous Variables}}, \overbrace{L^1, L^2, L^3,....., L^J}^{\text{Continuous Variables}}, \overbrace{M^1, M^2, M^3,....., M^J}^{\text{Continuous Variables}}, \overbrace{H^1, H^2, H^3,....., H^J}^{\text{Continuous Variables}},$$

$$\overbrace{Y^2, Y^3,....., Y^J}^{\text{Binary Variables}}, \overbrace{Z^1, Z^2, Z^3,....., Z^J}^{\text{Binary Variables}}, \overbrace{W_1^1, W_2^1,....., W_q^1, W_1^2, W_2^2,....., W_q^2, W_1^3, W_2^3,....., W_q^3 ...., W_q^j}^{\text{Continuous Variables}}\Big]$$

In the above structure, the first *J* variables correspond to *X*, the next three sets of *J* variables correspond to *L*, *M* and *H*. All these 4*J* variables are continuous decision variables. This is followed by the 2*J* binary variables corresponding to *Y* and *Z*. Thus the length of the decision vector is 6*J* with the last set of 2*J* variables being binary variables and corresponds to the value used in *intcon* in *intlinprog*. In the case of production with by-products, this structure has to be appropriately modified to incorporate the continuous variables corresponding to W_q^J .